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Some Notes on Aberration. By H. H. Turner, D.Sc. F.R.S., Savilian Professor.

1. It may be stated at the outset that the net result of the following discussion is to come back to the starting-point. It is the account of a train of thought, in the course of which a new departure was suggested more than once, only to prove misleading; and the only reason for printing the account is that it may possibly save others from some of the pitfalls which were noticed (generally by falling into them). It may seem presumptuous to think that others will not be more wary; but on at least one occasion I have found the opinion of others, familiar with the subject, to agree with mine, although this has ultimately proved erroneous. I do not wish, by saying this, to shirk responsibility for the mistakes made, but only to strengthen the case for printing this record of certain seductive errors and their exposure.

2. The keynote of the discussion is the following question, to which my attention was originally drawn by reading a most interesting chapter in Mr. N. R. Campbell's book, *Modern Electrical Theory*, Camb. Un. Press, 1907.

Does the correction for aberration in the position of an observed heavenly body P depend on the velocity of the observer *relative to* P, or upon his absolute velocity in space?

Our instinct is to reject the second alternative, because our knowledge of all motion can only be relative. Even the introduction of the notion of the ether, which enables us to talk of the Earth's motion "relatively to the ether," instead of the "absolute motion in space," is not altogether satisfactory. But if aberration depends on the relative motion of observer and star, into which the motion of the star enters just as much as that of the Earth, how is it that we correct for the motion of the Earth in its orbit, and neglect the motion of the star which may also be moving in an orbit? At first sight it certainly seems as though we are concerned specially with the absolute motion of the Earth, or something very like it.

3. But the difficulty is to devise a test which shall distinguish between the two hypotheses. This looks at first sight an easy matter, but it is not so, as will presently appear. The following test suggested itself to me, and was judged sound by others:—

Let A C B, fig. 1, be the equatorial section of the Sun; A and B the positions of a spot when it is just appearing and just disappearing; C its position on the centre of the disc. If aberration depends on relative velocity (*i.e.* on the velocity of the source of light just as much as on that of the recipient, only in the reverse direction of course), then the velocity of the spot across the line of sight at C should displace it aberrationally, while at A and B there would be no effect of that kind. Hence, if the heliographic longitude of the spot observed at C be compared with the mean of those at A and B,

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it will be found to be displaced. To assess the amount, we may compare with the 20″ due to the Earth's annual motion. The velocity of C is at a radius 1/200 that of the Earth's orbit, but C revolves in 25 days instead of 365; hence the amount should be 15 × 20″/200 = 1″·5 approximately.

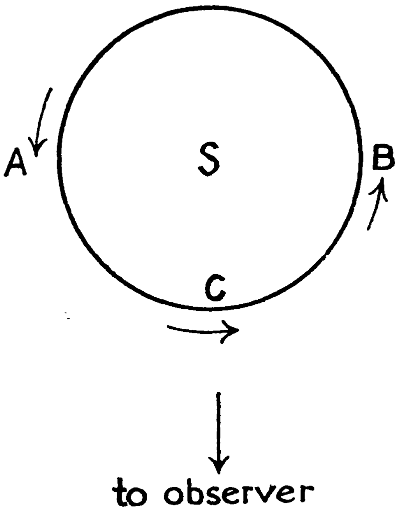

Fig. 1.

4. This is the effect to the observer, and is independent of his distance, just as the aberration correction for the stars is independent of their distance. When the observer is on the Earth, 1″·5 corresponds to 200 × 1″·5 = 5′ = 0°·08 of solar longitude. Now this is a small quantity, but my experience of the accuracy of the Greenwich sun-spot measures led me to think that it could be detected in the mean of a large number of observations; and as support was forthcoming for the view that the result would answer the question of § 2, the computations were undertaken in the Computing Bureau of the I.U.S.R. The longitudes of a large number of spots, as observed in the centre of the disc, were compared with the longitudes found for the same spots near the limb. It was not, of course, possible to use measures of the spots actually in the positions A and B of fig. 1, but positions near these were available. and in practice they were distant 65° from the central meridian. [To get good values of the longitudes at these points the means of values at 75°, 65°, and 55° were taken, by a sufficiently approximate process of interpolation.] The mean of the observed longitudes at +65° and −65° from the central meridian was then compared with that on the central meridian [also obtained from three consecutive days.] Only those spots were used which travelled completely across the disc; but all such spots as were available in the thirty-two years 1874–1905 were used.

5. The results are as follows, the quantity tabulated being the excess of the observed longitude in the centre of the disc over that near the edges.

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Excess of observed Longitude on central Meridian over that at ±65° *from centre for Greenwich spot-groups* 1874–1905.

Latitudes of Spots.	No. of Spots.	Observed Excess.
over − 25°	20	− 0°·43
− 20 to − 25	41	+ 0·15
− 15 to − 20	97	− 0·06
− 10 to − 15	103	− 0·14
− 0 to − 10	98	− 0·04
+ 0 to + 10	74	− 0·19
+ 10 to + 15	125	+ 0·01
+ 15 to + 20	57	− 0·25
+ 20 to + 25	31	− 0·24
over + 25	11	+ 0·16
Total	657	− 0·10 simple mean.
		− 0·09 weighted ,,

Weighted mean for spots between + 15° and − 15° of latitude − 0·08.

6. Since the velocity of the source should throw the object *back* (that of the Earth throws it apparently *forward*), the sign of the result accords with an aberration effect. As regards the amount, we must correct the quantity found above for the fact that outer observations were not on, but only near the limb, so that we have got only a portion of the total effect.

The reducing factor depends both on the latitude and on the distance (65°) in longitude from the central meridian; but the effect of the latitude is small, and it will be sufficient for our present purpose to take the factor as $(1 - \cos 65^{\circ}) = \cdot 58$; so that for the total aberration effect we should find (from the weighted mean)

$$-0^{\circ}\cdot 09/\cdot 58 = -0^{\circ}\cdot 15.$$

This is nearly double the amount computed above (0°·083), but is of the same order of magnitude, and seemed to afford some confirmation of the view that the velocity of the source of light enters into the aberration effect.

7. But while this work was being done, it occurred to me that there were serious difficulties attending the view that the velocity of the source of light could produce an aberration effect when we apply the principle to the stars. Aberration is an angular displacement independent of the distance of the body observed, whereas the actual movements of the stars are diminished by distance. Consequently, in the case of double stars, we should find the aberration effect increasing in importance relatively to the apparent size of the orbit as the orbit was more and more remote.

8. Suppose, for instance, there was a binary composed of two equal stars revolving in a year at the distance separating the Earth

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and Sun, in a plane through the line of sight. Then, when they were actually passing through the line of sight, they would be displaced by 20″ in opposite directions, or separated to the extent of 40″ to our vision (as seen some time subsequently). There is no visual binary with a period so short as a year, but there are three with periods under 20 years, and their maximum separations are all under 1″. Even if we ascribe the whole of this separation to the aberrational effect, we must attribute very small velocities, and therefore very small masses, to the components, and this does not accord with other known facts. Still greater difficulties occur in the case of the spectroscopic binaries which have known linear velocities in the line of sight, implying transverse velocities of similar magnitudes, and yet the components are not seen separated. It would seem that the velocity of the source of light cannot therefore enter into the aberration.

9. A more general difficulty was pointed out by a correspondent after consideration of the above instances. Accepting the result that the centre of the Sun's disc is displaced by 1″·5 to an observer at any distance, it follows that if the Sun were at a stellar distance its light would be spread over an arc 1″·5 in length, instead of being a stellar point. It is doubtful whether we should see it at all in that case, and, as my correspondent remarked, the only stars we should see would be those which do not rotate.

10. We seemed, therefore, to be in the position that a definite test applied to the Sun supported the view that the motion of the source *does* enter into the aberration; while the application of this principle to the stars requires an almost unthinkable interpretation of observed facts.

11. I now proceed to show that both of these arguments are fallacious. Let us first take the simple case of the Earth and a planet, where the motions are known relatively to the Sun, and the

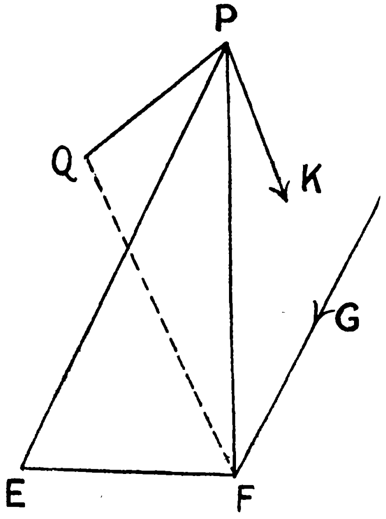

Fig. 2.

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time of light transmission is so small that the paths of the bodies may be taken as straight for the interval. Let P (fig. 2) be the position of the planet when the light leaves it at time T, and E the position of the Earth. The light reaches the Earth at a time *t*, when the Earth is at F and the planet has moved to Q, so that the light travels along the straight line PF. But on reaching the Earth at F it *appears* to come in the direction GF, owing to the aberration effect discovered by Bradley, which depends on compounding the velocities of the Earth and light; and since EF and PF are proportional to these velocities, GF is parallel to PE. Hence at the time T the planet actually was in the direction EP from the Earth in which it *appears* to be at the time *t*, and we get the well-known principle of "antedating" the observation by the interval *t*—T.

12. Now in this procedure we take no account of the velocity PQ of the planet, and it is therefore at first sight difficult to believe that anything can depend on the relative velocity of the Earth and planet. But let us now consider the state of things at the other end P, and see how the velocity of the planet enters into the question. The light which reaches the Earth leaves the planet in the direction PF; but just as this direction does not coincide with that in which it *appears* to reach the Earth, so, owing to the planet's velocity, it does not coincide with that in which it *appears* to leave the planet. If we wish to receive the light at F, we must point a telescope in the direction FG; and so, if we wish to transmit the light at P, we must point the discharging gun in the direction PK. The gun must fire with the velocity of light, and a better metaphor can be obtained by keeping to the telescope. To send light to the Earth, we may conceive that there is on the planet a telescope with a source of light in its focus which sends light through the object-glass to the Earth in strictly parallel rays. This telescope must be pointed, not in the direction PF in which the light actually travels, but in the direction PK, which is easily seen to be parallel to QF, by the composition of the velocities of the planet and light, just as FG is parallel to EP by the composition of the velocities of the Earth and light.

13. We can now see what happens when we use the *relative* velocity of Earth and planet in correcting for aberration, instead of the velocity of the Earth alone.

If we correct the observed direction GF for the velocity of the Earth alone, we get the direction PF.

If we further correct this direction PF for the velocity of the planet, we get the direction PK, which may be called the observed direction of transmission.

But it is easily seen that the velocity of the planet is applied in the opposite direction, so that the total correction is of the form earth minus planet, and thus depends directly on the relative velocity of Earth and planet.

Further, the direction PK is parallel to QF, which is the direction in which the planet actually lies from the Earth at the moment of observation *t*.

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Hence by applying to the observation a correction for aberration depending on the relative velocity of the Earth and planet, we get the actual position of the planet at the moment of observation. Thus the view that the aberration correction depends on relative velocity is completely justified.

14. We have then two alternatives in procedure:—

(α) We may accept the observation as it stands, as representing the position of the planet from the Earth at time T when the light started.

(β) We may correct the observation for the relative velocity of the Earth and planet, and we then obtain the position of the planet at time t when the observation is actually made.

15. These alternatives (with a third to be presently noticed) are at least as old as Gauss, and have been repeated from the *Theoria Motus* into various text-books, for the references to which I am indebted to Mr. H. C. Plummer. But in no case that we have been able to find is the point brought out that in (β) we use essentially the *relative* motion of Earth and planet, which is a conception of importance physically. Take, for instance, the following version of (β) from Chauvenet, vol. i. p. 642.

> "II. The true place and distance being known for the time t, we compute the reduction t—T. Thus by means of the diurnal motion of the planet (in longitude and latitude, or in right ascension and declination) we can reduce the true place from the time t to the time T; and the true place thus found is the apparent place at the time t."

This is a clear statement of the computational method by which the principle of (β) is put into practice; but the method of statement obscures the principle itself, and would scarcely convey to the physicist who is unfamiliar with our computational procedure that the net result was to apply an aberration depending on the relative velocity of Earth on planet. Consequently this essential point has been somewhat lost sight of, and I therefore venture to recall attention to it.

16. The third alternative mentioned by Gauss and his successors makes use of the line P F in fig. 2, the position of the planet at time T, as seen from the Earth at time t. Now it seems worth while to remark on the essential difference between P F and the lines P E and Q F which are used above in (α) and (β) respectively. The points P and E represent simultaneous positions of planet and Earth; and the direction P E is thus unaffected by the motion of the whole system, whereas P F depends on that motion. To bring out this point, let us make the simple supposition that the whole solar system happens to be travelling in space with a velocity equal and opposite to that of the planet, so that P is really at rest in space, or in the ether. Then we must substitute for fig. 2 a new construction, as in fig. 3. The Earth now moves along E K, compounded of E F and F K, equal to Q P. Hence the light travels, not along P F, but along P K. Its path depends in fact upon the assumed velocity of the solar system as a whole. But E P, the

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position of P from E at time T, and its parallel K G, the apparent direction at time *t*, remain unaffected, being parallel to their old directions in fig. 2. And P K in fig. 3, the position of the planet from the Earth at time *t*, is parallel to the corresponding Q F in fig. 2, and is now coincident with P K in fig. 2. Hence the alternatives (α) and (β) above, which use simultaneous positions, and are independent of the velocity of the system as a whole, differ essentially from the third, which uses the position of one body at time T and the other at time *t*, and thus depends on the motion of the system in the meantime. As between (α) and (β), the latter brings out the fact that the aberration depends on the velocity of the observer relatively to the source. This is indeed the proper aberration correction to obtain the position of the body at the moment of observation, though the greater simplicity of the alternative (α) causes it to be chosen as a practical rule.

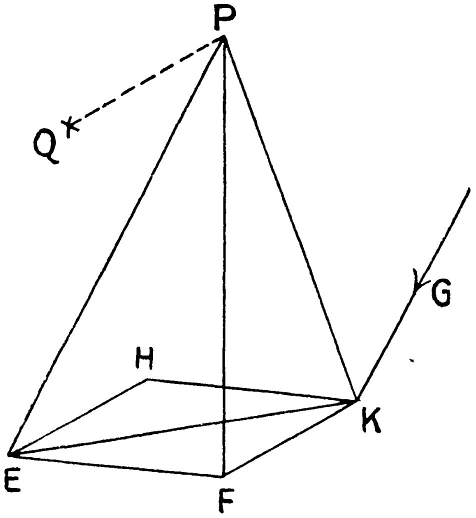

Fig. 3.

[Mr. Plummer remarked, after this paper was in type, that fig. 3 brings out clearly the function of the relative motion. For this relative motion of Earth and planet is the diagonal E K, and by applying it to E P we get P K, which is the result obtained in § 13 in two steps.]

17. Wherein, then, lies the fallacy in the argument about the sun-spot? It would appear that we have justified it by this reasoning, at any rate when we select the alternative (β). And yet, using alternative (α), we see that we can obtain equally true results by antedating each photograph by about 8 minutes (the time taken by light to reach us from the Sun), which will make no appreciable difference in the relative daily positions. It is true that this light-time is not quite the same for all parts of the Sun's surface; the limbs are further from us than the centre of the disc, and this does involve a correction which has not been hitherto applied; we

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will presently return to it, and show that it is too small to be sensible. Neglecting it for the present, let G H A B C D in fig. 4 represent the Sun's disc as seen from the Earth E. Then an observation made when the spot is actually at B gives us its true direction, as seen from E, 8 minutes earlier, when it actually was

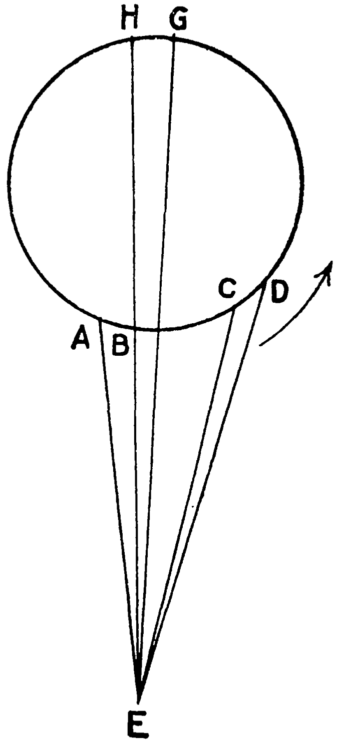

FIG. 4.

at A. Similarly, when the spot is actually at D we observe it at C; when actually at H we observe it at G; and the arcs A B, C D, G H are all equal if we neglect the small correction above mentioned. Taking the light-time as 8 minutes, and the time of a rotation of the Sun as 25 days, each of these little arcs is in degrees

$$\frac{8 \times 360T}{25 \times 24 \times 60} = 0^\circ\cdot08,$$

which is the quantity calculated in § 4. But we see, further, that this correction *when expressed in solar longitude* is applicable equally in all positions of the spot, and not specially to those in the centre of the disc. The velocity across the line of sight, on which the aberration effect depends, no doubt diminishes as we pass from the centre of the disc to the limbs, the reducing factor being the cosine of the longitude from the central meridian; but when we convert the measured position into solar longitude, we multiply by the secant of this same angle, which cancels out the cosine and leaves us a constant effect. The result found in § 6

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cannot, therefore, be due to a neglected aberration correction, and the explanation of it must be sought elsewhere.

18. As regards the small correction for the distance of the limbs beyond the centre of the disc, the time taken by light to traverse the Sun's radius is $2\frac{1}{2}$ seconds only, and the effect on the longitude is $0^{\circ}\cdot004$. This correction is theoretically applicable to solar photographs, but it is clearly quite insensible in the present state of observation.

19. We now come to the case of a star when the time T taken by the light to travel from star to Earth may be very large—thousands of years perhaps—and the Earth's path cannot be regarded as a straight line, as drawn in fig. 2. We can, however, re-interpret fig. 2 as follows.

P is still the position of the star when the light leaves it.

F is still the position of the Earth when the light is received.

G F is still the direction in which the light seems to come.

E F is still the direction of the Earth's motion at the moment of observation.

But E is no longer the point where the Earth actually was when the light started. It is the point where it *would have been* if it had been travelling since the time T with the velocity it has at the moment of observation.

And consequently E P is the direction in which the star P lay, from this fictitious Earth, at the time when the light started.

20. Now it is inconvenient to refer the positions of P to such a fictitious Earth; for this fiction describes a large orbit in space. Let F_1 F_2 F_3 in fig. 5 represent the actual orbit ot the Earth about the Sun S. Then an observation made at F_1 refers the star to a fictitious Earth E_1, E_1 F_1 being the distance the Earth would travel in many years: an observation at F_2 refers to E_2, and so on. The variation in the Earth's velocity causes the fictitious Earth to describe a huge orbit E_1 E_2 E_3. This fictitious Earth is consequently an inconvenient point of reference, and we accordingly do not use

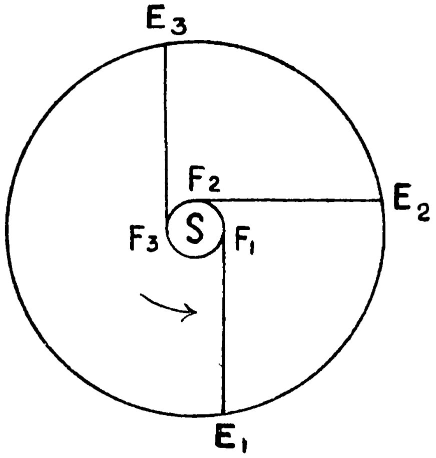

Fig. 5.

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it. We correct the observations for the variable part of the Earth's velocity in the well-known manner, and the observations are then referred to the Sun.

21. But the Sun is also moving, and we have still to consider the effect of his movement in space. The Sun now takes the place of the Earth in fig. 2: the observation being made at F_1 (and corrected, as above, for that part of the aberration which depends on the Earth's annual motion), we get the position in which the star P lay from the Sun at a previous time T, if the Sun's velocity has been uniform during this time. If it has not been uniform, the observation is referred to a fictitious Sun, which would have occupied the position E, obtained by producing the Sun's path backwards for a time T. Does any inconvenience arise from the use of this fictitious Sun, as from that of the fictitious Earth? Only if the velocity of the actual Sun changes; for then the fictitious Sun will sweep out a huge orbit in space, just as the fictitious Earth did.

22. Now we do not yet know whether the velocity of our Sun in space has changed during the period of astronomical observations, but there is a possibility that we may be in a position to detect such a change. A remarkable duplicity of the stellar universe has recently been discovered by Kapteyn, and developed by Eddington. There are some features about the phenomenon which would be more intelligible if the duplicity originated with the Sun rather than with the stars, *e.g.* the curious uniformity in all parts of the sky. Now if the Sun's velocity in space has changed in any way during the period of observation, then the place of any star is virtually referred during the time of observation to a fictitious Sun which is sweeping out a huge orbit in space. If this orbit were the same for all stars, it would be compounded with the actual orbit of the Sun, and we could not separate the two. But the radius of the fictitious Sun's orbit differs for every star, depending on the light-time for each. Hence we should get a duplicity in the parallactic motions. Another way of regarding the matter is this: a change in the Sun's position in space gives a parallactic displacement of the stars, the nearest being most affected. A change in the Sun's velocity will displace the stars in a manner which is in some respects similar to the parallactic, but is independent of the distances of the stars. We might thus conceivably get a duplex effect such as has been observed. On making inquiry of Mr. Eddington whether this was a possibility, I learnt that the same idea had been broached by Mr. Stratton some time ago, and rejected on examination. I hope the details of this examination will be published, as the point is of fundamental importance, and it seems to me still possible that the conclusion is open to revision. Mr. Eddington has kindly promised to go into the matter again.

23. So far, we have considered stellar aberration from the point of view of alternative (α) of § 14. But now let us turn to (β), which has some advantages, especially in offering us positions referred to the time of observation rather than to an epoch in the past—sometimes the very distant past. Our rule is that if we

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correct for the whole aberration, *i.e.* if we use the velocity of the Earth relatively to the star, we get the position in which the star *now is*, on a certain hypothesis. The hypothesis is, that it has been moving ever since the light started, with the velocity it then possessed. But since many of the stars conspicuously change their velocity, we find ourselves again in receipt of information cast in an inconvenient form. *Mutatis mutandis*, we may now take fig. 5 to represent the case of a double star. The little orbit $F_3 F_2 F_1$ may represent the orbit of the companion round its primary S, described now in the reverse direction. Suppose light starts at F_3. Then when it reaches the Earth, if we correct the observation for the full aberration, we get the information that the companion would have been at E_3 if it had persisted ever since with the velocity it had when the light started. If we now make a subsequent observation on light starting from F_2, we learn that the companion would similarly have now been at E_2 if it had persisted with the velocity of F_2, and so on. We see how the real orbit is replaced by a fictitious one of much greater dimensions, as was noticed in § 8; but we also see that this larger orbit has no real existence, and conveys to us little of value. The difficulty of § 9 is almost further removed from the domain of reality. We there consider the movement of the surface of a rotating body, which is thereby expanded enormously by aberration. The aberration correction is quite legitimate; but it gives us, *not* an appearance that we actually see (or fail to see), but a totally fictitious body that would have existed if every point of the rotating surface had persisted in a linear path with uniform velocity ever since the light started. Such a body would naturally be huge; but it is a pure mathematical abstraction.

24. Consequently we do not in practice use the fictitious orbit, or the fictitious rotating body. The information that the revolving companion of a double star *would have been* at such and such a point, *if* it had moved with uniform velocity for a long time, does not much interest us; and we seek something which does interest us by correcting for that part of the aberration which depends on its orbital velocity. But we have already applied this correction once, as part of the total correction for relative velocity of Earth and star; and it is easily seen that the second application will be in the opposite direction to the first, and will accordingly annul it. We are thus left with the correction for the velocity of the Earth alone; and thus we arrive at the answer to the question put in § 2, viz.—"how is it that we correct for the motion of the Earth in its orbit, and neglect the motion of the star which may also be moving in an orbit?" The answer is that the result will be in an inconvenient form if we use the whole relative velocity, and to get it in a convenient form we must annul the orbital velocity of the star.

25. In conclusion, it seems worthy of remark that there are one or two minute irregularities in the Sun's motion which our present practice (unless I am mistaken) does not take into account, and

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which may become sensible in modern refinements. In correcting the places of the stars, the aberration correction is made by the two terms—

$$A = k \cos \omega \cos \odot \qquad B = k \sin \odot$$

These depend on the longitude of the Sun relatively to the Earth, including all the perturbations. But they cannot contain any velocities common to Earth and Sun, such as, for instance, the minute rocking of the solar system by Jupiter. As Jupiter revolves, the whole solar system describes a minute orbit round the C.G., and there is a corresponding aberration. The amount is small, the coefficient being about $0''\cdot008$; but such a correction may be sensible in very refined investigations.

26. Again, there is a small term of about the same amount due to the rocking of the Earth by the Moon. I do not think it is included in the N.A. formulæ, although terms of smaller magnitude are retained—(*e.g.* in D we have the terms

$$+0''\cdot0008 \cos 2\Gamma' - 0''\cdot0027 \cos (3\odot - \Gamma). \text{ etc. etc.}).$$

Such terms only come into the observations of stars; planetary observations are fully corrected by the antedating method, if the time of transmission be correctly computed.

Note on the Calculation of Stellar Aberration.

By H. C. Plummer.

The following brief note is suggested by the concluding paragraphs of Professor Turner's paper on Aberration (p. 413), which I have been allowed to see in manuscript. He has pointed out that the usual method of calculating stellar aberration allows only partially for the effect of perturbations of the Earth's orbit. Where great accuracy is required, it may be well, at least as a check, to base the calculation directly on the use of the solar ephemeris.

The rectangular equatorial coordinates of the Sun are (X, Y, Z). Hence the components of the Earth's velocity relative to the Sun are $(-X', -Y', -Z')$, where X', Y', Z' can be derived from X, Y, Z by mechanical differentiation. Consider the rectangular directions (1) O S of the star; (2) O A perpendicular to O S, and such that the plane S O A contains the pole P; (3) O B perpendicular to both O S and O A. The components of the Earth's velocity vector in these directions are—

$$\begin{aligned}
u &= -\ (X' \cos \alpha \cos \delta + Y' \sin \alpha \cos \delta + Z' \sin \delta).\\
v &= -\ (-X' \cos \alpha \sin \delta - Y' \sin \alpha \sin \delta + Z' \cos \delta).\\
w &= -\ (-X' \sin \alpha + Y' \cos \alpha).
\end{aligned}$$

The first, u, is the reduction to the Sun of an observed radial